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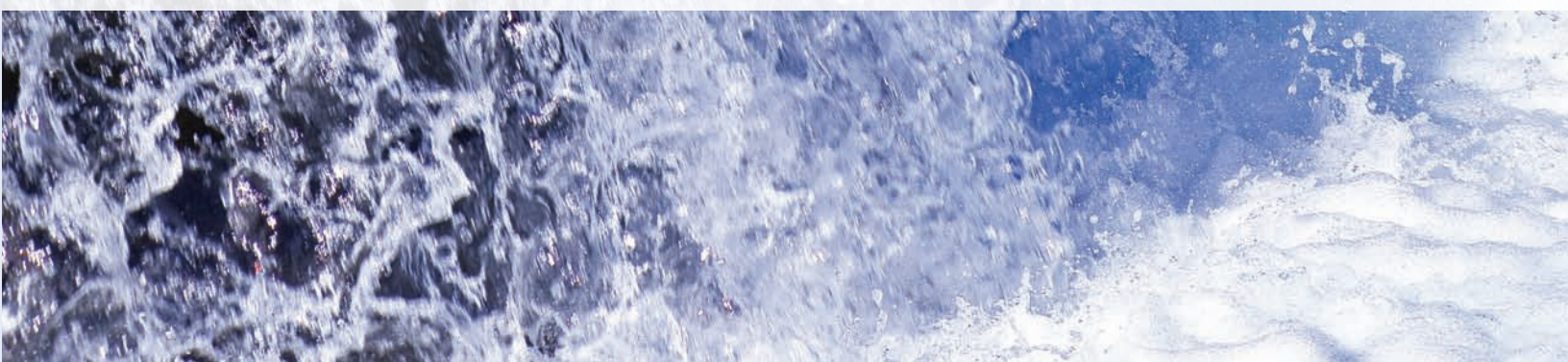


Calculus

for the Life Sciences

SECOND EDITION

Raymond N. Greenwell • Nathan P. Ritchey • Margaret L. Lial



ALWAYS LEARNING

PEARSON

Calculus for the Life Sciences

SECOND EDITION
GLOBAL EDITION

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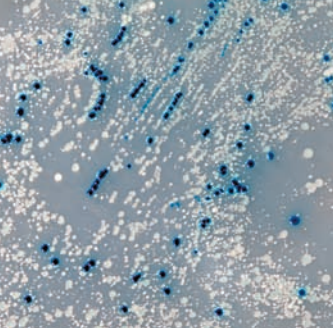
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	Absorbing Markov Chains		



Preface

Calculus for the Life Sciences emphasizes those aspects of calculus most relevant to the life sciences. The application examples and exercises are drawn predominately from biology, medicine, ecology, and other life sciences. A prerequisite of two years of high school algebra is assumed.

Our Approach

Our main goal is to present applied calculus in a concise and meaningful way so that students can understand the full picture of the concepts they are learning and apply it to real-life situations. This is done through a variety of ways.

Focus on Applications Making this course meaningful to students is critical to their success. Applications of the mathematics are integrated throughout the text in the exposition, the examples, the exercise sets, and the supplementary resources. *Calculus for the Life Sciences* presents students with myriad opportunities to relate what they're learning to life science and career situations through the *Apply It* questions, the applied examples, and the *Extended Applications*. To get a sense of the breadth of applications presented, look at the Index of Applications in the back of the book or the extended list of sources of real-world data at the back of this text and on www.pearsonglobaleditions.com/greenwell.

Pedagogy to Support Students Students need careful explanations of the mathematics along with examples presented in a clear and consistent manner. Additionally, students and instructors should have a means to assess the basic prerequisite skills. This can now be done with the *Prerequisite Skills Diagnostic Test* located just before Chapter R. In addition, the students need a mechanism to check their understanding as they go and resources to help them remediate if necessary. *Calculus for the Life Sciences* has this support built into the pedagogy of the text through fully developed and annotated examples, *Your Turn* exercises, *For Review* references, and supplementary material.

Beyond the Textbook Students today take advantage of a variety of resources and delivery methods for instruction. As such, we have developed a robust MyMathLab® course for *Calculus for the Life Sciences*. MyMathLab has a well-established and well-documented track record of helping students succeed in mathematics. The MyMathLab online course for *Calculus for the Life Sciences* contains exercises to challenge students and provides help when they need it. Students who learn best by seeing and hearing can view section- and example-level videos within MyMathLab. These and other resources are available to students as a unified and reliable tool for their success.

New to This Edition

Based on the authors' experience in the classroom along with feedback from many instructors across the country, the focus of this revision is to improve the clarity of the presentation and provide students with more opportunities to learn, practice, and apply what they've learned on their own. This is done both in the presentation of the content and in new features added to the text.

New Textbook Features

- **Exercises and examples have been updated** to reflect the latest data and to incorporate feedback received on the previous version of the text.
- **“Your Turn”** exercises following selected examples provide students with an easy way to stop and check their understanding of the skill or concept being presented. Answers are provided at the end of the section's exercises.

- The answers for each section (at the back of the book) now include a table listing the examples within the section that are most similar to each exercise.
- **Updated Chapter Reviews**
 - A list of important formulas and definitions has been added.
 - Review exercises now begin with Concept Check exercises—a series of true/false exercises designed to assess understanding of key ideas.
 - The answers for review exercises (at the back of the book) now include a table listing which section has examples most similar to each review exercise.
- The **Prerequisite Skills Diagnostic Test**, just prior to Chapter R, gives students and instructors an opportunity to assess students' skills on topics that are critical to success in this course. Answers reference specific review material in the text for **targeted remediation**.
- The updated design makes it **easier to identify graphing calculator and Excel® spreadsheet technology coverage** so instructors can more easily highlight (or skip) the material.

New MyMathLab® Course

- **Extensive exercise coverage**—choose from hundreds of assignable exercises to help you craft just the right assignments.
- **Diagnostic quizzes and personalized homework** provide individualized remediation for any gaps in prerequisite skills.
- **Video program** features example and section coverage.
- **Graphing calculator and Excel® spreadsheet** guidance for students.
- **Application exercises** within MyMathLab are now labeled by type of application.

New and Revised Content

Chapter 1 In Section 1.1, a new example has been added that models the prevalence of cigarette smoking in the United States, and then uses the model to make predictions. Section 1.2 has been revised, giving the formulas for the least squares line explicitly and making them more consistent with the formula for the correlation coefficient. In Section 1.4, an example on vaccination coverage, which illustrates how to derive a quadratic model, both by hand and with technology, has been added. In Section 1.5, material on identifying the degree of a polynomial has been rewritten as an example to better highlight the concept. An example on tuberculosis in the United States, which illustrates how to derive a cubic model, has also been added. Throughout the chapter, real-life exercises have been updated, and new exercises on topics such as cancer, diabetes, gender ratio, energy consumption, meat consumption, the demand for nurses, organic farming, and ideal partner height have been added.

Chapter 2 In Section 2.1, a new example on the surplus of food, which illustrates how to derive an exponential model, both by hand and with technology, has been added. In Section 2.4, an example using a trigonometric function to model the pressure on the eardrum has been included. A new Extended Application on power functions has been added. Throughout the chapter, real-life exercises have been updated and new exercises on topics such as the bald eagle population, minority population growth, carbon monoxide emissions, wind energy, metabolic rate, physician demand, and music have been added.

Chapter 3 In Section 3.1, the introduction of limits was completely revised. The opening discussion and example were transformed into a series of examples that progress through different limit scenarios: a function defined at the limit, a function undefined at that limit (a “hole” in the graph), a function defined at the limit but with a different value than the limit (a piecewise function), and then, finally, attempting to find a limit when one does not exist. New figures were added to illustrate the different scenarios. In Section 3.2, the definition and examples of continuity have been revised using a simple process to test for continuity. A medical device cost analysis has been added as an example. In Section 3.3, an example calculating the rate of change of the number of households with landlines has

been added. The opening discussion of Section 3.5, showing how to sketch the graph of the derivative given the graph of the original function, was rewritten as an example. An Extended Application on the modeling of drugs administered intravenously has been added. Throughout the chapter, real-life exercises have been updated, and new exercises on topics such as Alzheimer's disease, body mass index, and immigration have been added.

Chapter 4 The introduction to the chain rule was rewritten as an example in Section 4.3. In Section 4.4, a new example illustrates the use of a logistic function to develop a model for the weight of cactus wrens. An Extended Application on managing renewable resources has been added. Throughout the chapter, real-life exercises have been updated, and new exercises on topics such as tree growth, genetics, insect competition, whooping cranes, cholesterol, involuntal psychosis, radioactive iron, radioactive albumin, heat index, Jukes-Cantor distance, eardrum pressure, online learning, and minority populations have been added.

Chapter 5 Twenty-six new exercises were added throughout Chapter 5, nine of them applications based on scientific sources, such as three on foraging and two on cohesiveness, which involves the spacing between fish in a school or birds in a flock. There are now more trigonometric exercises in the chapter, along with an additional example involving trigonometry.

Chapter 6 Throughout Chapter 6, thirteen new exercises and three examples were added. Five of the new applications are based on scientific sources, such as the one on cancer cells and the one on honeycombs in Section 6.2 on Applications of the Derivative. In addition, there are now more trigonometric exercises in the chapter.

Chapter 7 The antiderivatives of the trigonometric functions have been moved to the first section so they can now be applied throughout the chapter. In addition, six exercises were added, some involving trigonometry and others involving integration using real data (such as those related to energy usage). The examples were improved by adding two new examples involving trigonometry and updating six involving real data.

Chapter 8 An example using tables to find trigonometric integrals has been included in Section 8.2. New graphics help illustrate the calculation of the volume of a solid of revolution in Section 8.3. Throughout the chapter, real-life exercises have been updated.

Chapter 9 In Chapter 9, seven of the new applications are based on scientific sources and fifteen new exercises were added, such as those on zooplankton growth, hypoxia (when the lungs receive inadequate oxygen), and Body Shape Index. In addition, many more trigonometric exercises are found throughout the chapter. The Extended Application has been revised to include an additional question involving technology as well as a group project option. Finally, material was added to justify the second derivative test for functions of two variables.

Chapter 10 In Section 10.1, a brief summary of solutions of two equations in two variables has been added. An alternate method for fraction-free Gauss-Jordan has been added to the first example. Section 10.5 has been revised, with the beginning discussion rewritten as examples on insect population growth, determinant calculations, and eigenvalue and corresponding eigenvector calculations. New exercises on finding the determinant of a matrix have also been included in this section. Throughout the chapter, real-life exercises have been updated, and new exercises on topics such as cancer, motorcycle helmets, calorie expenditure, basketball, and baseball have been added.

Chapter 11 The chapter opener was changed to relate to an exercise involving diseases in mouse populations. Material on equilibrium points and stability was added to Section 11.1. In addition, thirteen new exercises were added, six of them applications based on scientific sources. One of the new exercises compares the three population growth models discussed in the chapter: exponential, limited growth, and logistic. There are now more trigonometric exercises throughout the chapter. Finally, exercises were updated and an example, a Your Turn exercise, a For Review box, and a Technology Note were added.

Chapter 12 In Section 12.1, an example on endangered species has been added to illustrate set operations. In Section 12.2, an exam on student health has been included to show how Venn diagrams can be used to determine probabilities. In Section 12.3, the beginning discussion has been rewritten as an example of conditional probability. Examples on environmental inspections and on playing cards, both illustrating the product rule, have been added. The solution to the example on sensitivity and specificity has been rewritten for clarity. Challenging problems from actuarial exams have been included as exercises throughout this chapter as well.

Chapter 13 New material was added on the Poisson random variable and its connection to the exponential random variable. In addition, new material was inserted on a random sample, its mean, and the Central Limit Theorem. To support the new material as well as to better align with the existing content, four new examples were included, and thirty new exercises were added, of which four are applications based on scientific sources, such as the ones on plant growth and on the Hodgkin-Huxley model for excitable nerve cells.

Chapter 14 Chapter 14 in the last edition was available online for those who chose to cover discrete dynamical systems. With increased attention on this topic, the chapter has been heavily revised and supplemented to make for a robust topic in this course. Along with a new chapter opener, twelve new exercises were added to Section 14.1, six to Section 14.2, and thirteen to Section 14.3. A Chapter Review was added, with thirty exercises, two of them applications based on scientific sources. Finally, a new Extended Application was included, based on modeling the growth and decline of polio, as well as other life science situations that can be modeled by discrete dynamical systems.

Prerequisite Skills Diagnostic Test

The Prerequisite Skills Diagnostic Test gives students and instructors a means to assess the basic prerequisite skills needed to be successful in this course. In addition, the answers to the test include references to specific content in Chapter R as applicable so students can zero in on where they need improvement. Full solutions to the questions in this test are in Appendix A.

Your Turn Exercises

The **Your Turn** exercises, following selected examples, provide students with an easy way to quickly stop and check their understanding of the skill or concept being presented. Answers are provided at the end of the section's exercises.

More Applications and Exercises

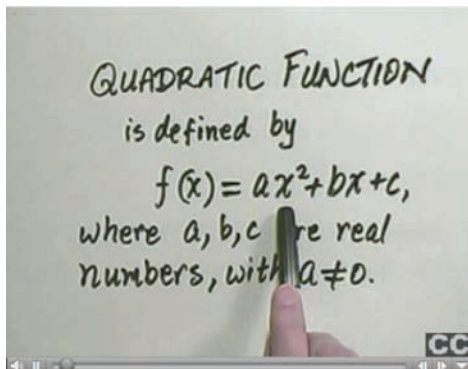
This text is used in large part because of the enormous amounts of real data contained in examples and exercises throughout. This second edition will not disappoint in this area. Every section of this text has new and updated examples, exercises and applications. The data has been updated in the applications whenever possible and new examples have been added altogether to provide additional guidance to students or new applications of the mathematics.

Coverage of Technology

Graphing calculator or **Microsoft Excel**[®] material is now set off to make it easier for instructors to use. Figures depicting graphing calculator screens have been redrawn to create a more accurate depiction of the math. In addition, this edition provides students with a transition to the new MathPrint[™] operating system of the TI-84+C through the Technology Notes, a new appendix, and the *Graphing Calculator Manual*. An *Excel*[®] *Spreadsheet Manual* is also available.

MyMathLab®

Available now with *Calculus for the Life Sciences* are the following resources within MyMathLab that will benefit students in this course.



- “Getting Ready for Applied Calculus” chapter covers basic prerequisite skills.
- Personalized Homework allows you to create homework assignments based on the results of student assessments.
- Premade “Getting Ready for Chapter X” quizzes are available for instructors to assign as a way to assess student preparedness for upcoming topics.
- Videos with example and section coverage.
- Hundreds of assignable exercises.
- Application labels within exercise sets (e.g., “Life Sciences”) make it easy to find types of applications appropriate for students with different interests.
- Additional graphing calculator and Excel spreadsheet help.

A detailed description of the overall capabilities of MyMathLab is provided on pages 13 and 14.

Source Lines

Sources for the exercises are now written in an abbreviated format within the actual exercise so that students immediately see that the problem comes from, or pulls data from, actual research or industry. The complete references are available on page S-1.

Continued Pedagogical Support

This second edition has undergone a significant revision to address the needs of today’s students. However, the signature writing style and features that make this text accessible to the average student have been maintained. Some of those signature features are the following:

Exposition and Examples

Nothing in a text is more important to a student than the examples and surrounding explanations. This text is known for its careful explanation of the mathematics and the fully developed examples. The examples often include explanatory comments in color, to the side of the worked-out examples, to help guide students through the steps. In addition, the authors frequently summarize key rules and formulas throughout the text to help students recognize the advantages of, or differences between, multiple techniques for solving a problem.

Multiple Methods of Solution

An emphasis remains on multiple representations of a topic, whenever possible, by examining each topic symbolically, numerically, graphically, and verbally, and by providing multiple methods of solution to various examples. Some of these alternative methods involve technology, while others represent a different way of performing a computation. These multiple representations give instructors the flexibility to emphasize the method they prefer and will ultimately help students better understand a topic.

Apply It

An **Apply It** question (previously named “Think About It”), typically at the start of a section, asks students to consider how to solve a real-life situation related to the math they are about to learn. The **Apply It** question is solved in an application within the section or the exercise set so students see it in context with the mathematics.

Extended Applications

In-depth applied exercises called “Extended Applications” are included at the end of every chapter to stimulate student interest and for use as group projects. These applications are also ideal for use in honors sections of the course.

For Review

For Review boxes are provided in the margin as appropriate, giving students just-in-time help with skills they should already know but may have forgotten. **For Review** comments sometimes include an explanation, while others refer students to earlier parts of the book for a more thorough review.

Caution and Notes

Common student difficulties and errors are highlighted under the heading “Caution” to keep students on track. In addition, important hints and asides are highlighted with the heading “Notes.”

Full-Color Graphics and Photography

To make this book lively and fun to read, full-color graphics have been included. As needed, color is used to enhance the exposition. More often, color is used to clarify the different parts of a figure or to highlight important aspects of an illustration.

Supplements

STUDENT RESOURCES

Graphing Calculator Manual (download only)

- By Victoria Baker, Nicholls State University
- Contains detailed instruction for using the TI-83/TI-83+/TI-84+ C.
- Instructions are organized by topic.
- Available in MyMathLab.

Excel® Spreadsheet Manual (download only)

- By Stela Pudar-Hozo, Indiana University—Northwest
- Contains detailed instructions for using Excel 2013.
- Instructions are organized by topic.
- Available in MyMathLab.

Video Lectures with Optional Captioning (download only)

- Videos with extensive example and section coverage, for student use at home or on campus.
- Ideal for distance learning or supplemental instruction.
- Available in MyMathLab.

INSTRUCTOR RESOURCES

Instructor’s Solutions Manual (download only)

- Provides instructors with detailed solutions to all section-level exercises, review exercises, and Extended Applications.
- Authored by Beverly Fusfield.
- Available to qualified instructors within MyMathLab or through the Pearson Instructor Resource Center, www.pearsonglobaleditions.com/greenwell

PowerPoint® Lecture Presentation

- Includes lecture content and key graphics from the book.
- Available to qualified instructors within MyMathLab or through the Pearson Instructor Resource Center, www.pearsonglobaleditions.com/greenwell

Media Resources

MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed.

- MyMathLab has a consistently positive impact on the quality of learning in higher education math instruction. MyMathLab can be successfully implemented in any environment—lab-based, hybrid, fully online, traditional—and demonstrates the quantifiable difference that integrated usage has on student retention, subsequent success, and overall achievement.
- MyMathLab’s comprehensive online gradebook automatically tracks your students’ results on tests, quizzes, homework, and in the study plan. You can use the gradebook to quickly intervene if your students have trouble, or to provide positive feedback on a job well done. The data within MyMathLab is easily exported to a variety of spreadsheet programs, such as Microsoft Excel. You can determine which points of data you want to export, and then analyze the results to determine success.

MyMathLab provides **engaging experiences** that personalize, stimulate, and measure learning for each student.

- **Personalized Learning:** MyMathLab offers features that support adaptive learning: personalized homework and the adaptive study plan. These features allow your students to work on what they need to learn when it makes the most sense, maximizing their potential for understanding and success.
- **Exercises:** The homework and practice exercises in MyMathLab correlate to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.
- **Chapter-Level, Just-in-Time Remediation:** The MyMathLab course for this text includes a short diagnostic, called “Getting Ready,” prior to each chapter to assess students’ prerequisite knowledge. This diagnostic can then be tied to Personalized Homework so that each individual student receives a homework assignment specific to his or her prerequisite skill needs.
- **Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at point-of-use.

And, MyMathLab comes from an **experienced partner** with educational expertise and an eye on the future.

- Knowing that you are using a Pearson product means knowing that you are using quality content. This means that our eTexts are accurate and our assessment tools work. It also means we are committed to making MyMathLab as accessible as possible. MyMathLab is compatible with the JAWS 12/13 screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. More information on this functionality is available at <http://mymathlab.com/accessibility>.
- Whether you are just getting started with MyMathLab, or have a question along the way, we’re here to help you learn about our technologies and how to incorporate them into your course.
- To learn more about how MyMathLab combines proven learning applications with powerful assessment and continuously adaptive capabilities, visit **www.mymathlab.com** or contact your Pearson representative.

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Dear Student,

Hello! The fact that you're reading this preface is good news. One of the keys to success in a math class is to read the book. Another is to answer all the questions correctly on your professor's tests. You've already started doing the first; doing the second may be more of a challenge, but by reading this book and working out the exercises, you'll be in a much stronger position to ace the tests. One last essential key to success is to go to class and actively participate.

You'll be happy to discover that we've provided the answers to the odd-numbered exercises in the back of the book. As you begin the exercises, you may be tempted to immediately look up the answer in the back of the book, and then figure out how to get that answer. It is an easy solution that has a consequence—you won't learn to do the exercises without that extra hint. Then, when you take a test, you will be forced to answer the questions without knowing what the answer is. Believe us, this is a lot harder! The learning comes from figuring out the exercises. Once you have an answer, look in the back and see if your answer agrees with ours. If it does, you're on the right path. If it doesn't, try to figure out what you did wrong. Once you've discovered your error, continue to work out more exercises to master the concept and skill.

Equations are a mathematician's way of expressing ideas in concise shorthand. The problem in reading mathematics is unpacking the shorthand. One useful technique is to read with paper and pencil in hand so you can work out calculations as you go along. When you are baffled, and you wonder, "How did they get that result?" try doing the calculation yourself and see what you get. You'll be amazed (or at least mildly satisfied) at how often that answers your question. Remember, math is not a spectator sport. You don't learn math by passively reading it or watching your professor. You learn mathematics by doing mathematics.

Finally, if there is anything you would like to see changed in the book, feel free to write to us at matrng@hofstra.edu or nritchey@edinboro.edu. We're constantly trying to make this book even better. If you'd like to know more about us, we have Web sites that we invite you to visit: <http://people.hofstra.edu/rgreenwell> and <http://users.edinboro.edu/nritchey>.

Ray Greenwell
Nate Ritchey

Prerequisite Skills Diagnostic Test

Below is a very brief test to help you recognize which, if any, prerequisite skills you may need to remediate in order to be successful in this course. After completing the test, check your answers in the back of the book. In addition to the answers, we have also provided the solutions to these problems in Appendix A. These solutions should help remind you how to solve the problems. For problems 5–26, the answers are followed by references to sections within Chapter R where you can find guidance on how to solve the problem and/or additional instruction. Addressing any weak prerequisite skills now will make a positive impact on your success as you progress through this course.

1. What percent of 50 is 10?
2. Simplify $\frac{13}{7} - \frac{2}{5}$.
3. Let x be the number of apples and y be the number of oranges. Write the following statement as an algebraic equation: “The total number of apples and oranges is 75.”
4. Let s be the number of students and p be the number of professors. Write the following statement as an algebraic equation: “There are at least four times as many students as professors.”
5. Solve for k : $7k + 8 = -4(3 - k)$.
6. Solve for x : $\frac{5}{8}x + \frac{1}{16}x = \frac{11}{16} + x$.
7. Write in interval notation: $-2 < x \leq 5$.
8. Using the variable x , write the following interval as an inequality: $(-\infty, -3]$.
9. Solve for y : $5(y - 2) + 1 \leq 7y + 8$.
10. Solve for p : $\frac{2}{3}(5p - 3) > \frac{3}{4}(2p + 1)$.
11. Carry out the operations and simplify: $(5y^2 - 6y - 4) - 2(3y^2 - 5y + 1)$.
12. Multiply out and simplify $(x^2 - 2x + 3)(x + 1)$.
13. Multiply out and simplify $(a - 2b)^2$.
14. Factor $3pq + 6p^2q + 9pq^2$.
15. Factor $3x^2 - x - 10$.
16. Perform the operation and simplify: $\frac{a^2 - 6a}{a^2 - 4} \cdot \frac{a - 2}{a}$.

17. Perform the operation and simplify: $\frac{x+3}{x^2-1} + \frac{2}{x^2+x}$.

18. Solve for x : $3x^2 + 4x = 1$.

19. Solve for z : $\frac{8z}{z+3} \leq 2$.

20. Simplify $\frac{4^{-1}(x^2y^3)^2}{x^{-2}y^5}$.

21. Simplify $\frac{4^{1/4}(p^{2/3}q^{-1/3})^{-1}}{4^{-1/4}p^{4/3}q^{4/3}}$.

22. Simplify as a single term without negative exponents: $k^{-1} - m^{-1}$.

23. Factor $(x^2 + 1)^{-1/2}(x + 2) + 3(x^2 + 1)^{1/2}$.

24. Simplify $\sqrt[3]{64b^6}$.

25. Rationalize the denominator: $\frac{2}{4 - \sqrt{10}}$.

26. Simplify $\sqrt{y^2 - 10y + 25}$.

R

Algebra Reference

- R.1 Polynomials
- R.2 Factoring
- R.3 Rational Expressions
- R.4 Equations
- R.5 Inequalities
- R.6 Exponents
- R.7 Radicals

In this chapter, we will review the most important topics in algebra. Knowing algebra is a fundamental prerequisite to success in higher mathematics. This algebra reference is designed for self-study; study it all at once or refer to it when needed throughout the course. Since this is a review, answers to all exercises are given in the answer section at the back of the book.



R.1 Polynomials

An expression such as $9p^4$ is a **term**; the number 9 is the **coefficient**, p is the **variable**, and 4 is the **exponent**. The expression p^4 means $p \cdot p \cdot p \cdot p$, while p^2 means $p \cdot p$, and so on. Terms having the same variable and the same exponent, such as $9x^4$ and $-3x^4$, are **like terms**. Terms that do not have both the same variable and the same exponent, such as m^2 and m^4 , are **unlike terms**.

A **polynomial** is a term or a finite sum of terms in which all variables have whole number exponents, and no variables appear in denominators. Examples of polynomials include

$$5x^4 + 2x^3 + 6x, \quad 8m^3 + 9m^2n - 6mn^2 + 3n^3, \quad 10p, \quad \text{and} \quad -9.$$

Order of Operations Algebra is a language, and you must be familiar with its rules to correctly interpret algebraic statements. The following order of operations have been agreed upon through centuries of usage.

- Expressions in **parentheses** are calculated first, working from the inside out. The numerator and denominator of a fraction are treated as expressions in parentheses.
- **Powers** are performed next, going from left to right.
- **Multiplication** and **division** are performed next, going from left to right.
- **Addition** and **subtraction** are performed last, going from left to right.

For example, in the expression $[6(x + 1)^2 + 3x - 22]^2$, suppose x has the value of 2. We would evaluate this as follows:

$$\begin{aligned} [6(2 + 1)^2 + 3(2) - 22]^2 &= [6(3)^2 + 3(2) - 22]^2 && \text{Evaluate the expression in the innermost parentheses.} \\ &= [6(9) + 3(2) - 22]^2 && \text{Evaluate 3 raised to a power.} \\ &= (54 + 6 - 22)^2 && \text{Perform the multiplication.} \\ &= (38)^2 && \text{Perform the addition and subtraction from left to right.} \\ &= 1444 && \text{Evaluate the power.} \end{aligned}$$

In the expression $\frac{x^2 + 3x + 6}{x + 6}$, suppose x has the value of 2. We would evaluate this as follows:

$$\begin{aligned} \frac{2^2 + 3(2) + 6}{2 + 6} &= \frac{16}{8} && \text{Evaluate the numerator and the denominator.} \\ &= 2 && \text{Simplify the fraction.} \end{aligned}$$

Adding and Subtracting Polynomials The following properties of real numbers are useful for performing operations on polynomials.

Properties of Real Numbers

For all real numbers a , b , and c :

- | | |
|---|-------------------------------|
| 1. $a + b = b + a$;
$ab = ba$; | Commutative properties |
| 2. $(a + b) + c = a + (b + c)$;
$(ab)c = a(bc)$; | Associative properties |
| 3. $a(b + c) = ab + ac$. | Distributive property |

EXAMPLE 1 Properties of Real Numbers

- (a) $2 + x = x + 2$ Commutative property of addition
 (b) $x \cdot 3 = 3x$ Commutative property of multiplication
 (c) $(7x)x = 7(x \cdot x) = 7x^2$ Associative property of multiplication
 (d) $3(x + 4) = 3x + 12$ Distributive property

One use of the distributive property is to add or subtract polynomials. Only like terms may be added or subtracted. For example,

$$12y^4 + 6y^4 = (12 + 6)y^4 = 18y^4,$$

and

$$-2m^2 + 8m^2 = (-2 + 8)m^2 = 6m^2,$$

but the polynomial $8y^4 + 2y^5$ cannot be further simplified. To subtract polynomials, we use the facts that $-(a + b) = -a - b$ and $-(a - b) = -a + b$. In the next example, we show how to add and subtract polynomials.

EXAMPLE 2 Adding and Subtracting Polynomials

Add or subtract as indicated.

(a) $(8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8)$

SOLUTION Combine like terms.

$$\begin{aligned} & (8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8) \\ &= (8x^3 + 3x^3) + (-4x^2 + 5x^2) + (6x - 9x) + 8 \\ &= 11x^3 + x^2 - 3x + 8 \end{aligned}$$

(b) $2(-4x^4 + 6x^3 - 9x^2 - 12) + 3(-3x^3 + 8x^2 - 11x + 7)$

SOLUTION Multiply each polynomial by the coefficient in front of the polynomial, and then combine terms as before.

$$\begin{aligned} & 2(-4x^4 + 6x^3 - 9x^2 - 12) + 3(-3x^3 + 8x^2 - 11x + 7) \\ &= -8x^4 + 12x^3 - 18x^2 - 24 - 9x^3 + 24x^2 - 33x + 21 \\ &= -8x^4 + 3x^3 + 6x^2 - 33x - 3 \end{aligned}$$

(c) $(2x^2 - 11x + 8) - (7x^2 - 6x + 2)$

SOLUTION Distributing the minus sign and combining like terms yields

$$\begin{aligned} & (2x^2 - 11x + 8) + (-7x^2 + 6x - 2) \\ &= -5x^2 - 5x + 6. \end{aligned}$$

TRY YOUR TURN 1

YOUR TURN 1 Perform the operation $3(x^2 - 4x - 5) - 4(3x^2 - 5x - 7)$.

Multiplying Polynomials The distributive property is also used to multiply polynomials, along with the fact that $a^m \cdot a^n = a^{m+n}$. For example,

$$x \cdot x = x^1 \cdot x^1 = x^{1+1} = x^2 \quad \text{and} \quad x^2 \cdot x^5 = x^{2+5} = x^7.$$

EXAMPLE 3 Multiplying Polynomials

Multiply.

(a) $8x(6x - 4)$

SOLUTION Using the distributive property yields

$$\begin{aligned} 8x(6x - 4) &= 8x(\mathbf{6x}) - 8x(\mathbf{4}) \\ &= 48x^2 - 32x. \end{aligned}$$

(b) $(3p - 2)(p^2 + 5p - 1)$

SOLUTION Using the distributive property yields

$$\begin{aligned}
 &(3p - 2)(p^2 + 5p - 1) \\
 &= 3p(p^2 + 5p - 1) - 2(p^2 + 5p - 1) \\
 &= 3p(p^2) + 3p(5p) + 3p(-1) - 2(p^2) - 2(5p) - 2(-1) \\
 &= 3p^3 + 15p^2 - 3p - 2p^2 - 10p + 2 \\
 &= 3p^3 + 13p^2 - 13p + 2.
 \end{aligned}$$

(c) $(x + 2)(x + 3)(x - 4)$

SOLUTION Multiplying the first two polynomials and then multiplying their product by the third polynomial yields

$$\begin{aligned}
 &(x + 2)(x + 3)(x - 4) \\
 &= [(x + 2)(x + 3)](x - 4) \\
 &= (x^2 + 2x + 3x + 6)(x - 4) \\
 &= (x^2 + 5x + 6)(x - 4) \\
 &= x^3 + 5x^2 + 6x - 4x^2 - 20x - 24 \\
 &= x^3 + x^2 - 14x - 24.
 \end{aligned}$$

TRY YOUR TURN 2**YOUR TURN 2** Perform the operation $(3y + 2)(4y^2 - 2y - 5)$.

A **binomial** is a polynomial with exactly two terms, such as $2x + 1$ or $m + n$. When two binomials are multiplied, the FOIL method (First, Outer, Inner, Last) is used as a memory aid.

EXAMPLE 4 Multiplying PolynomialsFind $(2m - 5)(m + 4)$ using the FOIL method.**SOLUTION**

$$\begin{aligned}
 (2m - 5)(m + 4) &= \overset{\text{F}}{(2m)}(\overset{\text{O}}{m}) + \overset{\text{O}}{(2m)}(\overset{\text{I}}{4}) + \overset{\text{I}}{(-5)}(\overset{\text{L}}{m}) + \overset{\text{L}}{(-5)}(4) \\
 &= 2m^2 + 8m - 5m - 20 \\
 &= 2m^2 + 3m - 20
 \end{aligned}$$

EXAMPLE 5 Multiplying PolynomialsFind $(2k - 5m)^3$.**SOLUTION** Write $(2k - 5m)^3$ as $(2k - 5m)(2k - 5m)(2k - 5m)$. Then multiply the first two factors using FOIL.

$$\begin{aligned}
 (2k - 5m)(2k - 5m) &= 4k^2 - 10km - 10km + 25m^2 \\
 &= 4k^2 - 20km + 25m^2
 \end{aligned}$$

Now multiply this last result by $(2k - 5m)$ using the distributive property, as in Example 3(b).

$$\begin{aligned}
 &(4k^2 - 20km + 25m^2)(2k - 5m) \\
 &= 4k^2(2k - 5m) - 20km(2k - 5m) + 25m^2(2k - 5m) \\
 &= 8k^3 - 20k^2m - 40k^2m + 100km^2 + 50km^2 - 125m^3 \\
 &= 8k^3 - 60k^2m + 150km^2 - 125m^3 \quad \text{Combine like terms.}
 \end{aligned}$$

Notice in the first part of Example 5, when we multiplied $(2k - 5m)$ by itself, that the product of the square of a binomial is the square of the first term, $(2k)^2$, plus twice the product of the two terms, $(2)(2k)(-5m)$, plus the square of the last term, $(-5k)^2$.

CAUTION

Avoid the common error of writing $(x + y)^2 = x^2 + y^2$. As the first step of Example 5 shows, the square of a binomial has three terms, so

$$(x + y)^2 = x^2 + 2xy + y^2.$$

Furthermore, higher powers of a binomial also result in more than two terms. For example, verify by multiplication that

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.$$

Remember, for any value of $n \neq 1$,

$$(x + y)^n \neq x^n + y^n.$$

R.1 EXERCISES

Perform the indicated operations.

- $(2x^2 - 6x + 11) + (-3x^2 + 7x - 2)$
- $(-4y^2 - 3y + 8) - (2y^2 - 6y - 2)$
- $-6(2q^2 + 4q - 3) + 4(-q^2 + 7q - 3)$
- $2(3r^2 + 4r + 2) - 3(-r^2 + 4r - 5)$
- $(0.613x^2 - 4.215x + 0.892) - 0.47(2x^2 - 3x + 5)$
- $0.5(5r^2 + 3.2r - 6) - (1.7r^2 - 2r - 1.5)$
- $-9m(2m^2 + 3m - 1)$
- $6x(-2x^3 + 5x + 6)$
- $(3t - 2y)(3t + 5y)$
- $(9k + q)(2k - q)$
- $(2 - 3x)(2 + 3x)$
- $(6m + 5)(6m - 5)$
- $\left(\frac{2}{5}y + \frac{1}{8}z\right)\left(\frac{3}{5}y + \frac{1}{2}z\right)$
- $\left(\frac{3}{4}r - \frac{2}{3}s\right)\left(\frac{5}{4}r + \frac{1}{3}s\right)$

- $(3p - 1)(9p^2 + 3p + 1)$
- $(3p + 2)(5p^2 + p - 4)$
- $(2m + 1)(4m^2 - 2m + 1)$
- $(k + 2)(12k^3 - 3k^2 + k + 1)$
- $(x + y + z)(3x - 2y - z)$
- $(r + 2s - 3t)(2r - 2s + t)$
- $(x + 1)(x + 2)(x + 3)$
- $(x - 1)(x + 2)(x - 3)$
- $(x + 2)^2$
- $(2a - 4b)^2$
- $(x - 2y)^3$
- $(3x + y)^3$

YOUR TURN ANSWERS

- $-9x^2 + 8x + 13$
- $12y^3 + 2y^2 - 19y - 10$

R.2 Factoring

Multiplication of polynomials relies on the distributive property. The reverse process, where a polynomial is written as a product of other polynomials, is called **factoring**. For example, one way to factor the number 18 is to write it as the product $9 \cdot 2$; both 9 and 2 are **factors** of 18. Usually, only integers are used as factors of integers. The number 18 can also be written with three integer factors as $2 \cdot 3 \cdot 3$.

The Greatest Common Factor To factor the algebraic expression $15m + 45$, first note that both $15m$ and 45 are divisible by 15; $15m = 15 \cdot m$ and $45 = 15 \cdot 3$. By the distributive property,

$$15m + 45 = 15 \cdot m + 15 \cdot 3 = 15(m + 3).$$

Both 15 and $m + 3$ are factors of $15m + 45$. Since 15 divides into both terms of $15m + 45$ (and is the largest number that will do so), 15 is the **greatest common factor** for the polynomial $15m + 45$. The process of writing $15m + 45$ as $15(m + 3)$ is often called **factoring out** the greatest common factor.

EXAMPLE 1 Factoring

Factor out the greatest common factor.

(a) $12p - 18q$

SOLUTION Both $12p$ and $18q$ are divisible by 6. Therefore,

$$12p - 18q = 6 \cdot 2p - 6 \cdot 3q = 6(2p - 3q).$$

(b) $8x^3 - 9x^2 + 15x$

SOLUTION Each of these terms is divisible by x .

$$\begin{aligned} 8x^3 - 9x^2 + 15x &= (8x^2) \cdot x - (9x) \cdot x + 15 \cdot x \\ &= x(8x^2 - 9x + 15) \quad \text{or} \quad (8x^2 - 9x + 15)x \end{aligned}$$

YOUR TURN 1 Factor $4z^4 + 4z^3 + 18z^2$.**TRY YOUR TURN 1**

One can always check factorization by finding the product of the factors and comparing it to the original expression.

CAUTION When factoring out the greatest common factor in an expression like $2x^2 + x$, be careful to remember the 1 in the second term.

$$2x^2 + x = 2x^2 + 1x = x(2x + 1), \text{ not } x(2x).$$

Factoring Trinomials A polynomial that has no greatest common factor (other than 1) may still be factorable. For example, the polynomial $x^2 + 5x + 6$ can be factored as $(x + 2)(x + 3)$. To see that this is correct, find the product $(x + 2)(x + 3)$; you should get $x^2 + 5x + 6$. A polynomial such as this with three terms is called a **trinomial**. To factor the trinomial $x^2 + 5x + 6$, where the coefficient of x^2 is 1, we use FOIL backwards.**EXAMPLE 2** Factoring a TrinomialFactor $y^2 + 8y + 15$.**SOLUTION** Since the coefficient of y^2 is 1, factor by finding two numbers whose *product* is 15 and whose *sum* is 8. Since the constant and the middle term are positive, the numbers must both be positive. Begin by listing all pairs of positive integers having a product of 15. As you do this, also form the sum of each pair of numbers.

Products	Sums
$15 \cdot 1 = 15$	$15 + 1 = 16$
$5 \cdot 3 = 15$	$5 + 3 = 8$

The numbers 5 and 3 have a product of 15 and a sum of 8. Thus, $y^2 + 8y + 15$ factors as

$$y^2 + 8y + 15 = (y + 5)(y + 3).$$

The answer can also be written as $(y + 3)(y + 5)$.If the coefficient of the squared term is *not* 1, work as shown below.**EXAMPLE 3** Factoring a TrinomialFactor $4x^2 + 8xy - 5y^2$.**SOLUTION** The possible factors of $4x^2$ are $4x$ and x or $2x$ and $2x$; the possible factors of $-5y^2$ are $-5y$ and y or $5y$ and $-y$. Try various combinations of these factors until one works (if, indeed, any work). For example, try the product $(x + 5y)(4x - y)$.

$$\begin{aligned} (x + 5y)(4x - y) &= 4x^2 - xy + 20xy - 5y^2 \\ &= 4x^2 + 19xy - 5y^2 \end{aligned}$$

This product is not correct, so try another combination.

$$\begin{aligned}(2x - y)(2x + 5y) &= 4x^2 + 10xy - 2xy - 5y^2 \\ &= 4x^2 + 8xy - 5y^2\end{aligned}$$

Since this combination gives the correct polynomial,

$$4x^2 + 8xy - 5y^2 = (2x - y)(2x + 5y).$$

YOUR TURN 2 Factor
 $6a^2 + 5ab - 4b^2$.

TRY YOUR TURN 2

Special Factorizations Four special factorizations occur so often that they are listed here for future reference.

Special Factorizations

$x^2 - y^2 = (x + y)(x - y)$	Difference of two squares
$x^2 + 2xy + y^2 = (x + y)^2$	Perfect square
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$	Difference of two cubes
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$	Sum of two cubes

A polynomial that cannot be factored is called a **prime polynomial**.

EXAMPLE 4 Factoring Polynomials

Factor each polynomial, if possible.

- (a) $64p^2 - 49q^2 = (8p)^2 - (7q)^2 = (8p + 7q)(8p - 7q)$ Difference of two squares
- (b) $x^2 + 36$ is a prime polynomial.
- (c) $x^2 + 12x + 36 = (x + 6)^2$ Perfect square
- (d) $9y^2 - 24yz + 16z^2 = (3y - 4z)^2$ Perfect square
- (e) $y^3 - 8 = y^3 - 2^3 = (y - 2)(y^2 + 2y + 4)$ Difference of two cubes
- (f) $m^3 + 125 = m^3 + 5^3 = (m + 5)(m^2 - 5m + 25)$ Sum of two cubes
- (g) $8k^3 - 27z^3 = (2k)^3 - (3z)^3 = (2k - 3z)(4k^2 + 6kz + 9z^2)$ Difference of two cubes
- (h) $p^4 - 1 = (p^2 + 1)(p^2 - 1) = (p^2 + 1)(p + 1)(p - 1)$ Difference of two squares

CAUTION

In factoring, always look for a common factor first. Since $36x^2 - 4y^2$ has a common factor of 4,

$$36x^2 - 4y^2 = 4(9x^2 - y^2) = 4(3x + y)(3x - y).$$

It would be incomplete to factor it as

$$36x^2 - 4y^2 = (6x + 2y)(6x - 2y),$$

since each factor can be factored still further. To *factor* means to factor completely, so that each polynomial factor is prime.

R.2 EXERCISES

Factor each polynomial. If a polynomial cannot be factored, write *prime*. Factor out the greatest common factor as necessary.

- $7a^3 + 14a^2$
- $3y^3 + 24y^2 + 9y$

- $13p^4q^2 - 39p^3q + 26p^2q^2$
- $60m^4 - 120m^3n + 50m^2n^2$
- $m^2 - 5m - 14$
- $x^2 + 4x - 5$

7. $z^2 + 9z + 20$

8. $b^2 - 8b + 7$

9. $a^2 - 6ab + 5b^2$

10. $s^2 + 2st - 3t^2$

11. $y^2 - 4yz - 21z^2$

12. $3x^2 + 4x - 7$

13. $3a^2 + 10a + 7$

14. $15y^2 + y - 2$

15. $21m^2 + 13mn + 2n^2$

16. $6a^2 - 48a - 120$

17. $3m^3 + 12m^2 + 9m$

18. $4a^2 + 10a + 6$

19. $24a^4 + 10a^3b - 4a^2b^2$

20. $24x^4 + 36x^3y - 60x^2y^2$

21. $x^2 - 64$

23. $10x^2 - 160$

25. $z^2 + 14zy + 49y^2$

27. $9p^2 - 24p + 16$

29. $27r^3 - 64s^3$

31. $x^4 - y^4$

22. $9m^2 - 25$

24. $9x^2 + 64$

26. $s^2 - 10st + 25t^2$

28. $a^3 - 216$

30. $3m^3 + 375$

32. $16a^4 - 81b^4$

YOUR TURN ANSWERS

1. $2z^2(2z^2 + 2z + 9)$

2. $(2a - b)(3a + 4b)$

R.3

Rational Expressions

Many algebraic fractions are **rational expressions**, which are quotients of polynomials with nonzero denominators. Examples include

$$\frac{8}{x-1}, \quad \frac{3x^2 + 4x}{5x - 6}, \quad \text{and} \quad \frac{2y + 1}{y^2}.$$

Next, we summarize properties for working with rational expressions.

Properties of Rational Expressions

For all mathematical expressions P , Q , R , and S , with $Q \neq 0$ and $S \neq 0$:

$$\frac{P}{Q} = \frac{PS}{QS}$$

Fundamental property

$$\frac{P}{Q} + \frac{R}{Q} = \frac{P + R}{Q}$$

Addition

$$\frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}$$

Subtraction

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

Multiplication

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} \quad (R \neq 0)$$

Division

When writing a rational expression in lowest terms, we may need to use the fact that $\frac{a^m}{a^n} = a^{m-n}$. For example,

$$\frac{x^4}{3x} = \frac{1x^4}{3x} = \frac{1}{3} \cdot \frac{x^4}{x} = \frac{1}{3} \cdot x^{4-1} = \frac{1}{3}x^3 = \frac{x^3}{3}.$$

EXAMPLE 1 Reducing Rational Expressions

Write each rational expression in lowest terms, that is, reduce the expression as much as possible.

$$(a) \frac{8x + 16}{4} = \frac{8(x + 2)}{4} = \frac{4 \cdot 2(x + 2)}{4} = 2(x + 2)$$

Factor both the numerator and denominator in order to identify any common factors, which have a quotient of 1. The answer could also be written as $2x + 4$.

YOUR TURN 1 Write in lowest

terms $\frac{z^2 + 5z + 6}{2z^2 + 7z + 3}$.

$$(b) \frac{k^2 + 7k + 12}{k^2 + 2k - 3} = \frac{(k + 4)(k + 3)}{(k - 1)(k + 3)} = \frac{k + 4}{k - 1}$$

The answer cannot be further reduced.

TRY YOUR TURN 1

CAUTION

One of the most common errors in algebra involves incorrect use of the fundamental property of rational expressions. Only common *factors* may be divided or “canceled.” It is essential to factor rational expressions before writing them in lowest terms. In Example 1(b), for instance, it is not correct to “cancel” k^2 (or cancel k , or divide 12 by -3) because the additions and subtraction must be performed first. Here they cannot be performed, so it is not possible to divide. After factoring, however, the fundamental property can be used to write the expression in lowest terms.

EXAMPLE 2 Combining Rational Expressions

Perform each operation.

$$(a) \frac{3y + 9}{6} \cdot \frac{18}{5y + 15}$$

SOLUTION Factor where possible, then multiply numerators and denominators and reduce to lowest terms.

$$\begin{aligned} \frac{3y + 9}{6} \cdot \frac{18}{5y + 15} &= \frac{3(y + 3)}{6} \cdot \frac{18}{5(y + 3)} \\ &= \frac{3 \cdot 18(y + 3)}{6 \cdot 5(y + 3)} \\ &= \frac{3 \cdot \cancel{6} \cdot 3 \cdot \cancel{(y + 3)}}{\cancel{6} \cdot 5 \cdot \cancel{(y + 3)}} = \frac{3 \cdot 3}{5} = \frac{9}{5} \end{aligned}$$

$$(b) \frac{m^2 + 5m + 6}{m + 3} \cdot \frac{m}{m^2 + 3m + 2}$$

SOLUTION Factor where possible.

$$\begin{aligned} \frac{(m + 2)(m + 3)}{m + 3} \cdot \frac{m}{(m + 2)(m + 1)} \\ &= \frac{m \cdot \cancel{(m + 2)} \cdot \cancel{(m + 3)}}{\cancel{(m + 3)} \cdot \cancel{(m + 2)} \cdot (m + 1)} = \frac{m}{m + 1} \end{aligned}$$

$$(c) \frac{9p - 36}{12} \div \frac{5(p - 4)}{18}$$

SOLUTION Use the division property of rational expressions.

$$\begin{aligned} \frac{9p - 36}{12} \cdot \frac{18}{5(p - 4)} & \quad \text{Invert and multiply.} \\ &= \frac{9 \cdot \cancel{(p - 4)}}{\cancel{6} \cdot 2} \cdot \frac{\cancel{6} \cdot 3}{5 \cdot \cancel{(p - 4)}} = \frac{27}{10} \quad \text{Factor and reduce to lowest terms.} \end{aligned}$$

$$(d) \frac{4}{5k} - \frac{11}{5k}$$

SOLUTION As shown in the list of properties, to subtract two rational expressions that have the same denominators, subtract the numerators while keeping the same denominator.

$$\frac{4}{5k} - \frac{11}{5k} = \frac{4 - 11}{5k} = -\frac{7}{5k}$$

(e) $\frac{7}{p} + \frac{9}{2p} + \frac{1}{3p}$

SOLUTION These three fractions cannot be added until their denominators are the same. A **common denominator** into which p , $2p$, and $3p$ all divide is $6p$. Note that $12p$ is also a common denominator, but $6p$ is the **least common denominator**. Use the fundamental property to rewrite each rational expression with a denominator of $6p$.

$$\begin{aligned} \frac{7}{p} + \frac{9}{2p} + \frac{1}{3p} &= \frac{6 \cdot 7}{6 \cdot p} + \frac{3 \cdot 9}{3 \cdot 2p} + \frac{2 \cdot 1}{2 \cdot 3p} \\ &= \frac{42}{6p} + \frac{27}{6p} + \frac{2}{6p} \\ &= \frac{42 + 27 + 2}{6p} \\ &= \frac{71}{6p} \end{aligned}$$

(f) $\frac{x + 1}{x^2 + 5x + 6} - \frac{5x - 1}{x^2 - x - 12}$

SOLUTION To find the least common denominator, we first factor each denominator. Then we change each fraction so they all have the same denominator, being careful to multiply only by quotients that equal 1.

$$\begin{aligned} \frac{x + 1}{x^2 + 5x + 6} - \frac{5x - 1}{x^2 - x - 12} &= \frac{x + 1}{(x + 2)(x + 3)} - \frac{5x - 1}{(x + 3)(x - 4)} \\ &= \frac{x + 1}{(x + 2)(x + 3)} \cdot \frac{(x - 4)}{(x - 4)} - \frac{5x - 1}{(x + 3)(x - 4)} \cdot \frac{(x + 2)}{(x + 2)} \\ &= \frac{(x^2 - 3x - 4) - (5x^2 + 9x - 2)}{(x + 2)(x + 3)(x - 4)} \\ &= \frac{-4x^2 - 12x - 2}{(x + 2)(x + 3)(x - 4)} \\ &= \frac{-2(2x^2 + 6x + 1)}{(x + 2)(x + 3)(x - 4)} \end{aligned}$$

YOUR TURN 2 Perform each of the following operations.

(a) $\frac{z^2 + 5z + 6}{2z^2 - 5z - 3} \cdot \frac{2z^2 - z - 1}{z^2 + 2z - 3}$

(b) $\frac{a - 3}{a^2 + 3a + 2} + \frac{5a}{a^2 - 4}$

Because the numerator cannot be factored further, we leave our answer in this form. We could also multiply out the denominator, but factored form is usually more useful.

TRY YOUR TURN 2

R.3 EXERCISES

Write each rational expression in lowest terms.

1. $\frac{5v^2}{35v}$

2. $\frac{25p^3}{10p^2}$

3. $\frac{8k + 16}{9k + 18}$

4. $\frac{2(t - 15)}{(t - 15)(t + 2)}$

5. $\frac{4x^3 - 8x^2}{4x^2}$

6. $\frac{36y^2 + 72y}{9y}$

7. $\frac{m^2 - 4m + 4}{m^2 + m - 6}$

8. $\frac{r^2 - r - 6}{r^2 + r - 12}$

9. $\frac{3x^2 + 3x - 6}{x^2 - 4}$

10. $\frac{z^2 - 5z + 6}{z^2 - 4}$

11. $\frac{m^4 - 16}{4m^2 - 16}$

12. $\frac{6y^2 + 11y + 4}{3y^2 + 7y + 4}$

Perform the indicated operations.

13. $\frac{9k^2}{25} \cdot \frac{5}{3k}$

14. $\frac{15p^3}{9p^2} \div \frac{6p}{10p^2}$

15. $\frac{3a + 3b}{4c} \cdot \frac{12}{5(a + b)}$

16. $\frac{a - 3}{16} \div \frac{a - 3}{32}$

17. $\frac{2k - 16}{6} \div \frac{4k - 32}{3}$

18. $\frac{9y - 18}{6y + 12} \cdot \frac{3y + 6}{15y - 30}$

19. $\frac{4a + 12}{2a - 10} \div \frac{a^2 - 9}{a^2 - a - 20}$

20. $\frac{6r - 18}{9r^2 + 6r - 24} \cdot \frac{12r - 16}{4r - 12}$

21. $\frac{k^2 + 4k - 12}{k^2 + 10k + 24} \cdot \frac{k^2 + k - 12}{k^2 - 9}$

22. $\frac{m^2 + 3m + 2}{m^2 + 5m + 4} \div \frac{m^2 + 5m + 6}{m^2 + 10m + 24}$

23. $\frac{2m^2 - 5m - 12}{m^2 - 10m + 24} \div \frac{4m^2 - 9}{m^2 - 9m + 18}$

24. $\frac{4n^2 + 4n - 3}{6n^2 - n - 15} \cdot \frac{8n^2 + 32n + 30}{4n^2 + 16n + 15}$

25. $\frac{a + 1}{2} - \frac{a - 1}{2}$

26. $\frac{3}{p} + \frac{1}{2}$

27. $\frac{6}{5y} - \frac{3}{2}$

28. $\frac{1}{6m} + \frac{2}{5m} + \frac{4}{m}$

29. $\frac{1}{m - 1} + \frac{2}{m}$

30. $\frac{5}{2r + 3} - \frac{2}{r}$

31. $\frac{8}{3(a - 1)} + \frac{2}{a - 1}$

32. $\frac{2}{5(k - 2)} + \frac{3}{4(k - 2)}$

33. $\frac{4}{x^2 + 4x + 3} + \frac{3}{x^2 - x - 2}$

34. $\frac{y}{y^2 + 2y - 3} - \frac{1}{y^2 + 4y + 3}$

35. $\frac{3k}{2k^2 + 3k - 2} - \frac{2k}{2k^2 - 7k + 3}$

36. $\frac{4m}{3m^2 + 7m - 6} - \frac{m}{3m^2 - 14m + 8}$

37. $\frac{2}{a + 2} + \frac{1}{a} + \frac{a - 1}{a^2 + 2a}$

38. $\frac{5x + 2}{x^2 - 1} + \frac{3}{x^2 + x} - \frac{1}{x^2 - x}$

YOUR TURN ANSWERS

1. $(z + 2)/(2z + 1)$

2a. $(z + 2)/(z - 3)$

2b. $6(a^2 + 1)/[(a - 2)(a + 2)(a + 1)]$

R.4 Equations

Linear Equations Equations that can be written in the form $ax + b = 0$, where a and b are real numbers, with $a \neq 0$, are **linear equations**. Examples of linear equations include $5y + 9 = 16$, $8x = 4$, and $-3p + 5 = -8$. Equations that are *not* linear include absolute value equations such as $|x| = 4$. The following properties are used to solve linear equations.

Properties of Equality

For all real numbers a , b , and c :

1. **If $a = b$, then $a + c = b + c$.** **Addition property of equality**
(The same number may be added to both sides of an equation.)

2. **If $a = b$, then $ac = bc$.** **Multiplication property of equality**
(Both sides of an equation may be multiplied by the same number.)

EXAMPLE 1 Solving Linear Equations

Solve the following equations.

(a) $x - 2 = 3$

SOLUTION The goal is to isolate the variable. Using the addition property of equality yields

$$x - 2 + 2 = 3 + 2, \text{ or } x = 5.$$

(b) $\frac{x}{2} = 3$

SOLUTION Using the multiplication property of equality yields

$$2 \cdot \frac{x}{2} = 2 \cdot 3, \quad \text{or} \quad x = 6.$$

The following example shows how these properties are used to solve linear equations. The goal is to isolate the variable. The solutions should always be checked by substitution into the original equation.

EXAMPLE 2 Solving a Linear EquationSolve $2x - 5 + 8 = 3x + 2(2 - 3x)$.**SOLUTION**

$$\begin{aligned} 2x - 5 + 8 &= 3x + 4 - 6x && \text{Distributive property} \\ 2x + 3 &= -3x + 4 && \text{Combine like terms.} \\ 5x + 3 &= 4 && \text{Add } 3x \text{ to both sides.} \\ 5x &= 1 && \text{Add } -3 \text{ to both sides.} \\ x &= \frac{1}{5} && \text{Multiply both sides by } \frac{1}{5}. \end{aligned}$$

YOUR TURN 1 Solve $3x - 7 = 4(5x + 2) - 7x$.

Check by substituting in the original equation. The left side becomes $2(1/5) - 5 + 8$ and the right side becomes $3(1/5) + 2[2 - 3(1/5)]$. Verify that both of these expressions simplify to $17/5$.

TRY YOUR TURN 1

Quadratic Equations An equation with 2 as the highest exponent of the variable is a *quadratic equation*. A **quadratic equation** has the form $ax^2 + bx + c = 0$, where a , b , and c are real numbers and $a \neq 0$. A quadratic equation written in the form $ax^2 + bx + c = 0$ is said to be in **standard form**.

The simplest way to solve a quadratic equation, but one that is not always applicable, is by factoring. This method depends on the **zero-factor property**.

Zero-Factor PropertyIf a and b are real numbers, with $ab = 0$, then either

$$a = 0 \quad \text{or} \quad b = 0 \quad (\text{or both}).$$

EXAMPLE 3 Solving a Quadratic EquationSolve $6r^2 + 7r = 3$.**SOLUTION** First write the equation in standard form.

$$6r^2 + 7r - 3 = 0$$

Now factor $6r^2 + 7r - 3$ to get

$$(3r - 1)(2r + 3) = 0.$$

By the zero-factor property, the product $(3r - 1)(2r + 3)$ can equal 0 if and only if

$$3r - 1 = 0 \quad \text{or} \quad 2r + 3 = 0.$$